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Linear Control of a Two-Wheeled Self Balancing Autonomous Mobile Robot

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Abstract. This paper presents the control of the position of a Segway robot through a feedback of the state vector. We demonstrate that using this simple linear approach the Segway personal transportation can be stabilized. The approximate linearization is used in the nonlinear model for the design of the control law, advantages such as the easy implementation in an embedded system and the low calculation cost in processors has motivated choosing this approach. To validate the design of the control law, three parameters have been taken into account: settlement time, ability to reject external disturbances, and trajectory tracking.

Keywords. Segway, linear control, ESP32, robot operating system (ROS), full state feedback control.

1 Introduction

Mobile robotics is one of the sciences that has received attention in the industrial-academic and social fields. This is due in large part to the development of embedded computing systems, microelectromechanical systems MEMS sensors, devices with high energy storage and operating systems. Mobile robots have a wide niche of applications: tracking and transfer of objects, evasion of obstacles, cooperative environments, analysis and inspection, exploration of remote areas to mention some. In particular, land mobile robots are the research objective in this work.

These robots can be divided mainly by the mechanisms they use to move, generally divide them into wheels, tracks and legs. The simplest case of mobile robots is wheeled robots. Wheeled robots comprise one or more driven wheels and have optional passive or caster wheels and possibly steered wheels. The arrangement of the driven, passive and steered wheels gives different configurations of land mobile robots.

Today, balancing robots have gained popularity with the introduction of the commercial Segway vehicle. These robots have the ability to balance on their two wheels, making them excellent personal electric transport. Due to their great maneuverability, these robots allow people to be transported over short distances with

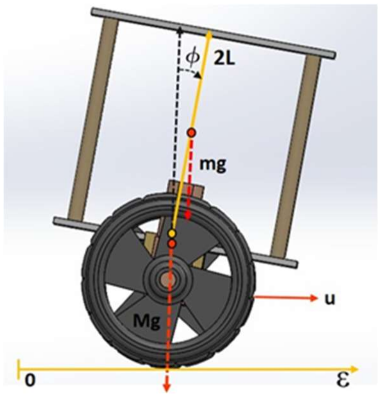

Fig. 1. Two-wheeled self-balancing.

a moderate speed, navigate over confined spaces indoors and outdoors, turn on their axis and in sharp corners, in addition to going through small unevennesses.

The physical balancing robot is an inverted pendulum with two independently driven motors, to allow for balancing, as well as driving straight and turning [1].

The balancing robot has two independent fixed motors to drive the wheels at each end, unlike the differential robots, the balancing robot lacks passive wheels, also the bar and the person on board makes the stabilization of the system more complicated.

In [3] linear LQR control is designed, it is implemented on an embedded FPGA-based system. The results show indoor experimental tests mounting stable robot control. In [4] a linear LQR control and an ISMC are designed, the mathematical model is based on the inverted pendulum on a mobile platform, the comparison between both algorithms is performed in simulation. In [5] a control based on a geometric PID is designed. This approach ensures almost global locally exponential stability of the upright motion of the Segway. The effectiveness of the control law is demonstrated through simulations against uncertainties.

In [6] a Segway with multidirectional shock absorbers between the vehicle motors and the main user platform are designed. The control scheme implemented is a PID controller when the vehicle is moving. Also, when the Segway is stationary, the PID controller ensures the smooth balancing of the vehicle. The state equations found from the mathematical modelling of the vehicle were used in MATLAB to calculate the optimum compensation constant values for the controller.

On the other hand, in [7] an optimal control method for the nonlinear system of Segway PT is proposed for reduces energy consumption and enhance the response speed of the system instead of classic PID controller. The controller showed the balance between speed response and control cost and the rejection of disturbance. Finally, in [8] a linear LQR control is implemented to control the Segway by adjusting parameters of controller gain. The results focus on the control algorithm which can adapt controller gain by driver weight.

The paper is structured as follows. In section 2 some mathematical preliminaries used along the document are presented, also the presentation of the model of the Segway Robot. In section 3 presents the control law design based on state feedback. In

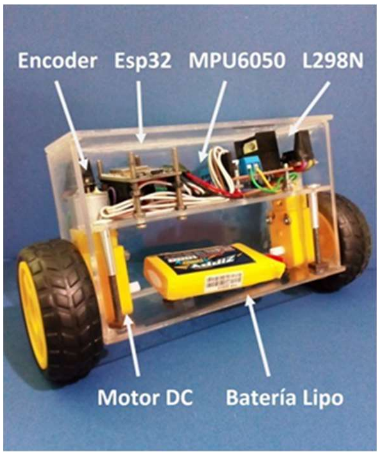

Fig. 2. Low-cost prototype of the inverted pendulum robot.

the 4 the simulation results are present which show the effectiveness of the proposed algorithm. In section 5 some conclusions and future work are presented.

2 Two-Wheeled Self-Balancing System Model

The segway robot model that we present is based on the inverted pendulum principle. The dynamics of the inverted pendulum has been the basis for bipedal robots, primary space propellers, and attitude control of small satellites.

The basic movements of the robot, driving forward-backward movement, driving sideways movement and rotate on its own axis.The forward-backward movement is achieved when the speed of both motors are increased or decreased by the same amount.

The sideways movement is achieved when the speed of the left motor is increased, while the speed of the right is decreased and vice versa. The rotate on its own axis is achieved when the speed of both motors are increased in the opposite direction.

The embedded system is composed of a 32-bit dual core ESP32 microcontroller with a 240 MHz clock frequency. This chip contains wireless communication modules such as WiFi and Bluetooth. The actuation system is made up of two - 5 V permanent magnet direct current motors.

The power system has a 1000 mAh 2s Zippy Lipo Battery, an MPU-6050 inertial unit and a L298N H-bridge as the power stage. Selecting the following variables (ξ, φ) and using the Euler-Lagrange formalism, the dynamics that describe the physics of the vehicle is obtained. The nonlinear model of the inverted pendulum is can be modeled as:

$$(m + M)\ddot{x} + (ml\cos\theta)\ddot{\theta} = -C_1\dot{x} + (ml\sin\theta)\ddot{\theta}^2 + u, \quad (1)$$

$$(ml\cos\theta)\ddot{x} + (J + ml^2)\ddot{\theta} = -C_2\dot{\theta} + (mgl\sin\theta), \quad (2)$$

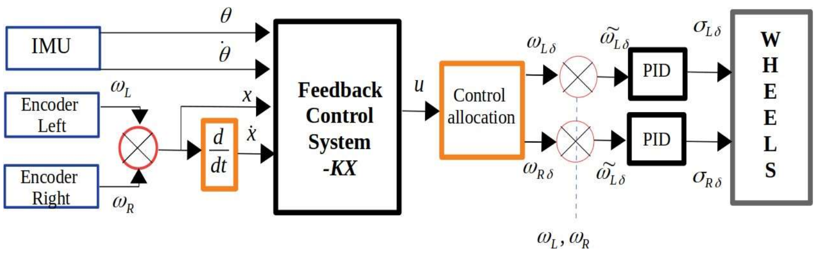

Fig. 3. Block diagram of the Segway control system.

where the state variables are defined as the displacement of the mobile x, the speed of the mobile as $\dot{x}$, as well as the angular displacement of the bar and the rider θ and its angular velocity $\dot{\theta}$.

The variable u represents the force that pushes the system in order to control it, therefore they are the signals from the motors.The constants J is the moment of inertia with respect to the center of gravity ofthe pendulum, l represents the length of the bar to the center of mass, M is the mass of the base of the Segway, m is the mass of the bar and the human, and $g$represents gravity. The constants C_1 and C_2 represent the coefficients of frictionof the rotary movement of the pendulum and of the linear movement of the base.For the mathematical control model, some restrictions have been considered:

- The center of gravity of the bar is at its geometric center.
- The coefficients of friction of the rotary movement of the pendulum and the linear movement of the base are negligible.
- The angular displacement θ is very small.

The Segway model for the design of the control algorithm is:

$$\ddot{x} = \frac{-m^2l^2g\theta+(J+ml^2)u}{\Delta}, \tag{3}$$

$$\ddot{\theta} = \frac{(M+m)(mgl\theta)-mlu}{\Delta}, \tag{4}$$

where $\Delta = (M+m)(J+ml^2) - m^2l^2$. Selecting as state variable $x_1 = x$, $x_2 = \dot{x}, x_3 = \theta$ and $x_4 = \dot{\theta}$. Consider the equilibrium point of this system in the upright vertical position:

$$[x_1\ x_2\ x_3\ x_4]^T = [x_d\ 0\ 0\ 0]^T \text{ further } u = 0, \tag{5}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{(J+ml^2)u-m^2l^2gx_3}{\Delta} \\ x_4 \\ \frac{-mlu+(M+m)mglx_3}{\Delta} \end{bmatrix}, \tag{6}$$

The approximate linearization of the system at the equilibrium point is:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & \frac{-m^2l^2g}{\Delta} & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & \frac{(M+m)mlg}{\Delta} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{J+ml^2}{\Delta}\\ 0\\ \frac{-ml}{\Delta}\end{bmatrix}u. \quad (7)$$

For the mathematical control model, some restrictions have been considered:

- The center of gravity of the bar is at its geometric center.
- The coefficients of friction of the rotary movement of the pendulum and thelinear movement of the base are negligible.

2.1 Propulsor Model

The system includes a brushless DC motor, ESC and propellers. First, givena constant voltage by the LIPO battery, the ESC generate an average voltage signal which is a function of the pulse width modulation PWM signal which will make the motor achieve steady-state speed. The above can be modeled as:

$$w_{ss} = C_R\sigma + w_b, \quad (8)$$

where C_R is a constant that depends on the battery voltage and the ESC, σ is PWM signal with a value between 1000 and 2000 indicate the duty cycles, and w_b is the angular speed which the motor reaches once the motor initializes. Because the motor needs some time to achieve the steady state speed Ω_{ss}, a second equation is generated to model the dynamic response. Generally, the dynamics of a brushless DC motor can be simplified as a first order equation:

$$w = \frac{1}{T_m s+1} w_{ss}, \quad (9)$$

Combining the Equations 9 and 8 get the complete propulsor model:

$$w = \frac{1}{T_m s + 1}(C_R\sigma + w_b\ . \quad (10)$$

3 Full State Feedback Control

The strategy to follow is to design a linear control law based on the state feedback for the lineal system eq. (7), so that it stabilizes the state variables to zero. Once the linear system has stabilized, the linear control law is applied tothe nonlinear system eq. (1) and eq. (2). This would ensure that the system is operating at its equilibrium point.

We verify that the linear system is controllable:

$$\det(C) = \det([B{:}\,AB{:}\,A^2B{:}\,A^3B]) \neq 0, \quad (11)$$

We propose the following control law:

$$u = -k_1x_1 - k_2x_2 - k_3x_3 - k_4x_4, \quad (12)$$

Propose that the linearized closed-loop system have its poles located at the roots of the following desired polynomial:

$$p_d(s) = (s + \alpha)^2(s^2 + 2\zeta w_n s + w_n^2). \quad (13)$$

where ζ is the damping factor and w_n is the undamped natural frequency corresponding to the pair of complex conjugated poles generating the second degree factor. Solving the system of equations:

2.2 Control Motor

Efficient BLDC motor speed control is required for the Segway robot. A quick and smooth response will produce high performance in position control.We assume that the angular speed of the brushless direct current motor is w, furthermore these motors use high frequency pulse width modulation (PWM) to control the motor voltage. The control objective is to design each motor a PWM signal command, such that $\lim_{t\to\infty} |w_d(t) - w(t)| = 0$. We propose a PID control to minimize the error signal $\breve{w} = w_d(t) - w(t)$:

$$\sigma = k_p\breve{w} + k_i \int \breve{w} + k_d\breve{w}. \quad (14)$$

The angular speed of the motors w can be obtained from the encoder, C_R and w_b are obtained through experimentation.

4 Simulation

The figure 3 shows the block diagram of the system. The system has two main sensors, an Inertial Measurement Unit (IMU) and an encoder on each BLDC motor. The IMU sensor estimates the angle θ using a built-in Kalman filter and the angular velocity $\dot{\theta}$. The encoders provide information on the angular speed of the wheel, with the angular speed we can calculate the position x and the speed of the base $\dot{x}$. The four state variables are fed back to the control algorithm and it is calculated u.

The control allocation block allows the control signal to be converted into the required angular velocity $w_{L\delta}$ and $w_{L\delta}$ for the motors. Lastly, a PID is necessary to ensure the desired angular velocity. The choiceof w_n and ζ is made to set the rise time of the system.

Typically ζ is chosen as 0.707, in order to achieve a damped response, w_n is chosen as $12 = {}^{rad}/_{s}$. The figures show the evolution of state variables of the non-linear system controlled by the feedback law based on approximate linearization in different conditions.The parameters for the simulation of the system are $M = 0.43,\ m = 0.16, l = 0.25, J = 0.0043, g = 9.81$ The vector values $K = [k_1 k_2 k_3 k_4]^T$, is obtained by solving the system of equations that is obtained by equating the coefficients of the desired polynomial term by term, k_1=-195.71,k_2=-78.28,k_3=-192.42,k_4=-36.05. In he figures 4, 5, 6 and 7 we try to reproduce the forward-backward command with the inclination of the bar by θ=23^o, that is to say that propose X_δ=[0 0 20 0]^T as equilibrium point for the

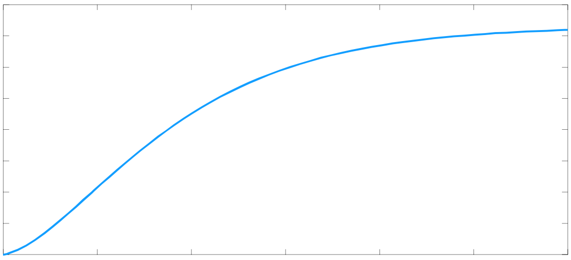

Fig. 4. Forward-backward movement, x evolution.

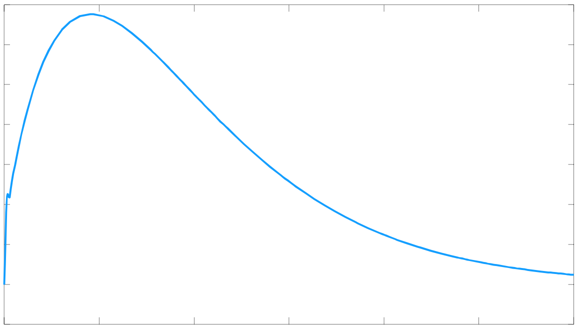

Fig. 5. Forward-backward movement, $\dot{x}$ evolution.

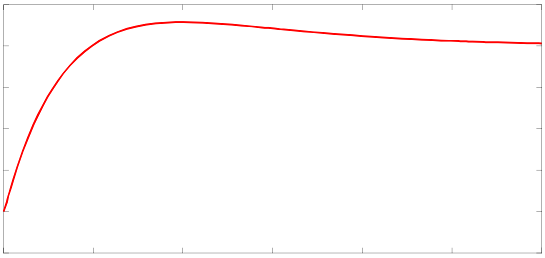

Fig. 6. Forward-backward movement, θ evolution.

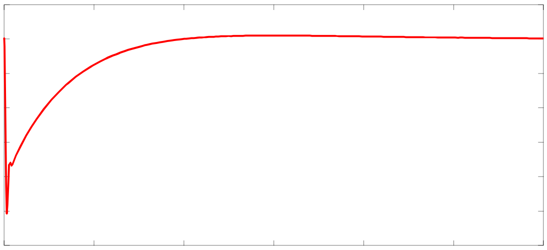

Fig. 7. Forward-backward movement, $\dot{\theta}$ evolution.

following initial conditions x(0)=[0 0 0 0]^T. As we can see in the images when tilting the bar by an angle θ, the base tries to compensate the bar to its vertical position, therefore its movement is towards forward.

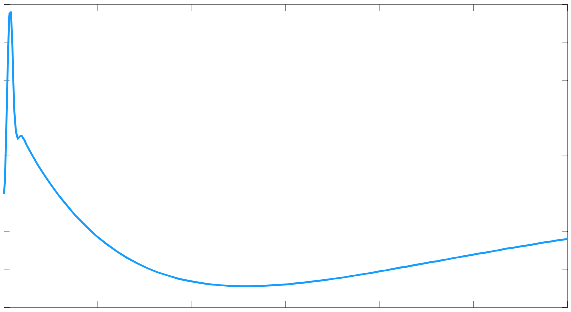

Fig. 8. Hover movement, x evolution.

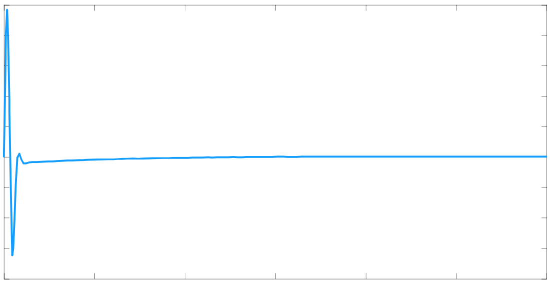

Fig. 9. Hover movement, $\dot{x}$ evolution.

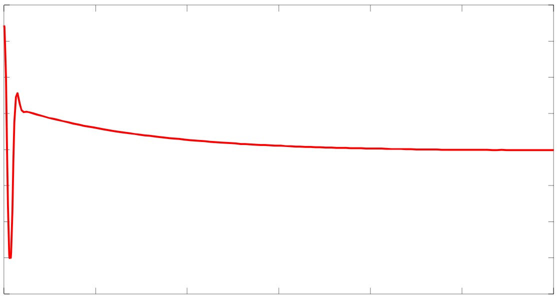

Fig. 10. Hover movement, θ evolution.

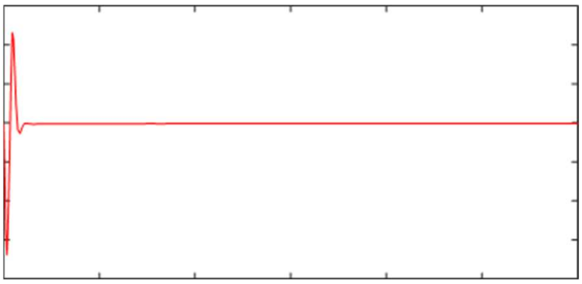

Fig. 11. Hover movement, $\dot{\theta}$ evolution.

The figures 8, 9, 10 and 11 show a hover movement while the robot bar is disturbed. The initial system condition is $x(0) = [0\ 0\ 35\ 0]^T$.

The figure 10 shows the position of the bar with an initial condition at 35^o can see how the bar stabilizes at zero due to the movement of the base see figure 8. Stabilization time is approximately 2.5 seconds, which is an excellent time for these systems.

5 Conclusions

It should be emphasized that the linearized system-based design applied to the nonlinear system at least around its equilibrium point may not stabilize the system if the disturbances move away from the equilibrium point, however, due to the nature of the Segway robot (small angles) the linear control law proposed by state feedback is stable enough for the stability of said robot. The design, instrumentation and implementation of the embedded system will represent a great challenge, an ARM microcontroller with an SBC is proposed for the embedded control system.

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